

Cosmological entropy and generalized second law of thermodynamics in $F(R, G)$ theory of gravity

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Abstract

We consider a spatially flat Friedmann-Lemaitre-Robertson-Walker space time and investigate the second law and the generalized second law of thermodynamics for apparent horizon in generalized modified Gauss Bonnet theory of gravity (whose action contains a general function of Gauss Bonnet invariant and the Ricci scalar: $F(R, G)$). By assuming that the apparent horizon is in thermal equilibrium with the matter inside it, conditions which must be satisfied by $F(R, G)$ are derived and elucidated through two examples: a quasi-de Sitter space-time and a universe with power law expansion.

1 Introduction

To explain the present accelerated expansion of the universe, which is confirmed by several astrophysical data [1], different models have been introduced. One of these models, dubbed as dark energy model, assumes that the universe is dominated by an exotic matter with negative pressure. This kind of matter violates the strong energy condition $P > -\frac{\rho}{3}$. Besides, models in which the Einstein theory of gravity is modified, have been also used to describe the present acceleration of the universe [2]. In these models there is no need for exotic matter with odd properties, but instead, the action contains general function of invariants obtained from the Riemann curvature tensor such as the Ricci scalar, R , or Gauss-Bonnet invariant term, G (such a model appears also in the low energy limit of heterotic string theory [3]). Consistency with other laws of physics (e.g. thermodynamics laws considered in this paper) and astrophysical data put many conditions on these modified theories.

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Thermodynamics aspects of general relativity and cosmology, such as validity of thermodynamics laws for cosmological horizons[4], thermodynamics of dark energy and so on have also been the subject of many studies in the past and recent years. The generalized second law (GSL) of thermodynamics in Einstein theory of gravity was investigated in [5], and was extended to the case of dark energy in [6]. The study of this law for the *future event horizon* in the simple modified theory of gravity whose action contains only a function of R or G , can be found in [7].

Recently, the generalized modified Gauss-Bonnet gravity, whose the action contains a general function of R and G ($F(R, G)$), has attracted more attention. Besides its stability, this is due to its ability to describe the present acceleration of the universe as well as the phantom divide line crossing and transition from acceleration to deceleration phases [8].

The scheme of the paper is as follows:

We consider the generalized modified Gauss-Bonnet theory of gravity in a spatially flat Friedman-Robertson-Walker (FRW) space time. As in this theory there may be many choices for $F(R, G)$, all leading to a same dynamics, GSL by putting some constraints on $F(R, G)$ can be considered as a test to choose a viable model. We study the thermodynamics second law and generalized second law of thermodynamics for the *apparent horizon* (there are many studies about the thermodynamics aspects of the apparent horizon, for a review see [9]). We will assume that the apparent horizon is in thermal equilibrium with the matter. We investigate the conditions obeyed by $F(R, G)$ to satisfy these thermodynamics laws and show that some of these conditions are in agreement with the other constraints required for dynamical stability of the model discussed before in the literature. We elucidate our results through two important cosmological examples: the quasi-de Sitter space-time which is a good candidate for the early stage and late time evolution of the universe, and a space-time with power law expansion filled by ordinary matter such as dust and radiation.

Units with $\hbar = c = G_N = k_B = 1$ are used in the paper.

2 Apparent horizon entropy and cosmological GSL in $F(R, G)$ model

2.1 preliminaries

$F(R, G)$ model of gravity is described by the action

$$S = \int \left(\frac{1}{16\pi} F(R, G) + \mathcal{L}_m \right) \sqrt{-g} d^4x, \quad (1)$$

where \mathcal{L}_m is the matter lagrangian density and $F(R, G)$ is a function of the Ricci scalar curvature and Gauss-Bonnet invariant defined by

$$G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}. \quad (2)$$

By varying the action with respect to the metric components $g_{\mu\nu}$, one obtains the (gravitational) field equations:

$$\begin{aligned} 8\pi T^{\mu\nu} = & g^{\mu\nu}\nabla^2 F_R - \nabla^\mu\nabla^\nu F_R - \frac{1}{2}g^{\mu\nu}F + R^{\mu\nu}F_R + 2g^{\mu\nu}R\nabla^2 F_G \\ & - 4R^{\mu\nu}\nabla^2 F_G - 2R(\nabla^\mu\nabla^\nu)F_G + 4R^{\mu\alpha}\nabla_\alpha\nabla^\nu F_G + 4R^{\nu\alpha}\nabla_\alpha\nabla^\mu F_G \\ & - 4g^{\mu\nu}R^{\alpha\beta}\nabla_\alpha\nabla_\beta F_G + 4R^{\mu\alpha\nu\beta}\nabla_\alpha\nabla_\beta F_G + 2RR^{\mu\nu}F_G - 4R^\mu_\alpha R^{\nu\beta}F_G \\ & + 2R^{\mu\alpha\beta\gamma}R^\nu_{\alpha\beta\gamma}F_G + 4R_{\alpha\beta}R^{\mu\alpha\beta\nu}F_G, \end{aligned} \quad (3)$$

where $F_R = \frac{\partial F(R, G)}{\partial R}$, $F_G = \frac{\partial F(R, G)}{\partial G}$ and so on, and $T^{\mu\nu}$ is the energy-momentum tensor of matter. We consider a spatially flat Friedmann-Robertson-Walker (FRW) space time in comoving coordinates

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2), \quad (4)$$

where $a(t)$ is the scale factor. In terms of the Hubble parameter $H = \frac{\dot{a}}{a}$, where the over dot indicates derivative with respect to the time t , we have

$$R = 6(\dot{H} + 2H^2), \quad G = 24H^2(\dot{H} + H^2). \quad (5)$$

Equation (3) gives the following equations

$$\begin{aligned} F_R\dot{H} &= -4\pi(P_m + \rho_m) + \frac{1}{2}(H\dot{F}_R - \ddot{F}_R + 4H^3\dot{F}_G - 8H\dot{H}\dot{F}_G \\ &\quad - 4H^2\ddot{F}_G), \\ F_R H^2 &= \frac{8\pi}{3}\rho + \frac{1}{6}(F_R R - F - 6H\dot{F}_R + GF_G - 24H^3\dot{F}_G). \end{aligned} \quad (6)$$

ρ_m and P_m are energy density and pressure of the matter component which behaves as a perfect fluid at large scale, satisfying the continuity equation

$$\dot{\rho}_m + 3H(P_m + \rho_m) = 0. \quad (7)$$

In the above, the matter ingredient may consist of various interacting components.

2.2 Apparent horizon entropy

In terms of the Hubble parameter, the apparent horizon radius is given by $R_h = \frac{1}{H}$. The entropy of this dynamical horizon, S_h , obtained by the Noether charge method [10] is given by

$$S_h = -\frac{1}{8}\int_{Horizon} \left(F_R \frac{\partial R}{\partial R_{\alpha\beta\gamma\rho}} + F_G \frac{\partial G}{\partial R_{\alpha\beta\gamma\rho}} \right) \varepsilon_{\alpha\beta}\varepsilon_{\gamma\rho} dA_h, \quad (8)$$

where $\varepsilon_{\mu\nu}$ are binormal vectors to horizon surface and dA_h is the differential surface element. Applying this result to the apparent horizon in the FRW space time yields:

$$S_h = \pi (H^{-2}F_R + 4F_G). \quad (9)$$

Dynamics of the horizon entropy is described by

$$\dot{S}_h = -2\pi H^{-3}\dot{H}F_R + \pi H^{-2}\dot{F}_R + 4\pi\dot{F}_G. \quad (10)$$

Note that a linear term in Gauss-Bonnet invariant in F , may change the horizon entropy but has no influence on its time derivative.

For a de Sitter space-time, $\dot{H} = 0$, and therefore $\dot{S}_h = 0$. If $F(R, G) = R$, for $\dot{H} < (>)0$, we have $\dot{S}_h > (<)0$. So in a super-accelerated universe the second law of thermodynamics does not hold for the apparent horizon in Einstein theory of gravity. This may not be true in modified theories of gravity. To show this, let us consider a quasi-de Sitter space-time which depends mildly on time:

$$H = H_0 + H_0^2\epsilon t + \mathcal{O}(\epsilon^2); \quad \epsilon = \frac{\dot{H}}{H^2}; \quad \dot{\epsilon} = \mathcal{O}(\epsilon^2), \quad (11)$$

where $|\epsilon| \ll 1$. By expanding R and G around de Sitter point, H_0 , up to order $\mathcal{O}(\epsilon^2)$, as

$$\begin{aligned} R &= 6H^2(2 + \epsilon) = 12H_0^2 + 6H_0^2(1 + 4H_0t)\epsilon + \mathcal{O}(\epsilon^2) \\ G &= 24H^4(1 + \epsilon) = 24H_0^4 + 24H_0^4(1 + 4H_0t)\epsilon + \mathcal{O}(\epsilon^2), \end{aligned} \quad (12)$$

and by using

$$\begin{aligned} \dot{F}_R &= (F_{RR}(H_0) + F_{RRR}(H_0)(R - R_0) + F_{RRG}(H_0)(G - G_0))\dot{R} \\ &\quad + (F_{RG}(H_0) + F_{RRG}(H_0)(R - R_0) + F_{RGG}(H_0)(G - G_0))\dot{G}, \\ \dot{F}_G &= (F_{GR}(H_0) + F_{GRR}(H_0)(R - R_0) + F_{RGG}(H_0)(G - G_0))\dot{R} \\ &\quad + (F_{GG}(H_0) + F_{RGG}(H_0)(R - R_0) + F_{GGG}(H_0)(G - G_0))\dot{G}, \end{aligned} \quad (13)$$

where $G_0 = G(H_0)$ and $R_0 = R(H_0)$, we conclude that the thermodynamics second law for the horizon, $\dot{S}_h \geq 0$, implies

$$(192F_{GG}(H_0)H_0^6 + 96F_{RG}(H_0)H_0^4 + 12F_{RR}(H_0)H_0^2 - F_R(H_0))\epsilon \geq 0. \quad (14)$$

Equation (6) results in that in a de Sitter space-time $P_m + \rho_m = 0$. Hence from the continuity equation we find out that ρ is a constant: $\rho_m(H_0) = \Lambda \in \mathbb{R}^+$. The solution with $\rho_m(H_0) = 0$ is dubbed as de Sitter vacuum solution. In general, F can be any function that satisfies

$$16\pi\rho_m(H_0) + 6H_0^2F_R(H_0) + 24H_0^4F_G(H_0) - F(H_0) = 0 \quad (15)$$

provided that (15) has a positive root $H_0 > 0$.

In contrast to the Einstein theory of gravity, depending on the form of F , (14) may be respected. To see this Let us choose a model with $F_R(H_0) < 0$. For the vacuum de Sitter solution ($\rho_m(H_0) \approx 0$), the stability of $F(R, G)$ model requires [11]

$$\frac{F_R(H_0)}{12H_0^2 F_{RR}(H_0) + 96H_0^4 F_{RG}(H_0) + 192H_0^6 F_{GG}(H_0)} > 1. \quad (16)$$

By comparing (14) and (16), we find out that in a stable model, thermodynamics second law for the apparent horizon holds whenever $\dot{H} > 0$.

2.3 GSL

Now let us consider time evolution of entropy of the matter inside the horizon, denoted by S_{in} , and study the GSL which states that the sum of the horizon entropy and the matter entropy is not decreasing in time:

$$\dot{S}_{tot} = \dot{S}_{in} + \dot{S}_h \geq 0. \quad (17)$$

In this way the horizon entropy is related to the information behind it. Hence we can consider the entropy of the universe as the sum of the entropy of the matter inside the horizon, and the horizon entropy.

In the absence of matter, or when the role of matter is negligible, GSL reduces to thermodynamics second law for the apparent horizon discussed before. From the first law of thermodynamics:

$$dE = TdS_{in} - PdV, \quad (18)$$

where $V = \frac{4\pi}{3H^3}$, and the continuity equation, we obtain

$$\dot{S}_{in} = -8\pi^2 H^{-5} (\dot{H} + H^2) (P_m + \rho_m). \quad (19)$$

The matter and the horizon are in thermal equilibrium and, in analogy with black hole thermodynamics, we have taken the horizon temperature $T = \frac{1}{2\pi H}$. For an accelerated expansion $(\dot{H} + H^2) > 0$ and for ordinary matter whose the pressure is positive, we have $\dot{S}_{in} < 0$. So a necessary condition for the GSL to hold is $\dot{S}_h > 0$.

In the case that the universe is dominated by a barotropic perfect fluid (at large scale), $P_m = w_m \rho_m$, the continuity equation yields

$$\rho = \tilde{\rho}_0 a^{-3\gamma}, \quad (20)$$

and the generalized second law can be written as

$$-2\pi H^{-3} \dot{H} F_R + \pi H^{-2} \dot{F}_R + 4\pi \dot{F}_G - 8\pi^2 H^{-5} (H^2 + \dot{H}) \gamma \tilde{\rho}_0 a^{-3\gamma} \geq 0. \quad (21)$$

We have defined $\gamma = w_m + 1$. But in more general cases the matter component may consist of several barotropic fluids and w_m is not necessarily

a constant, and obtaining an analytical solution for matter density is not straightforward. In these cases it is more convenient to use (6), and write (19) as

$$\dot{S}_{in} = \pi H^{-5}(\dot{H} + H^2)(2F_R\dot{H} - H\dot{F}_R + \ddot{F}_R - 4H^3\dot{F}_G + 8H\dot{H}\dot{F}_G + 4H^2\ddot{F}_G). \quad (22)$$

Hence GSL reads

$$\begin{aligned} \dot{S}_{tot} = & 2\pi H^{-5}\dot{H}^2 F_R - \pi H^{-4}\dot{H}\dot{F}_R + 4\pi H^{-4}\dot{H}(H^2 + 2\dot{H})\dot{F}_G \\ & + 4\pi H^{-3}(H^2 + \dot{H})\ddot{F}_G + \pi H^{-5}(\dot{H} + H^2)\ddot{F}_R \geq 0. \end{aligned} \quad (23)$$

For a de Sitter space-time the universe undergoes an adiabatic reversible expansion. Although the horizon entropy is not necessarily increasing in Einstein theory of gravity, but GSL holds generally in this theory: $\dot{S}_{tot} = 2\pi H^{-5}\dot{H}^2 \geq 0$.

To elucidate the rôle of matter entropy in GSL, let us reconsider the quasi-de Sitter space-time. As an interesting result the expression linear in ϵ in \dot{S}_h is cancelled out with the corresponding expression in \dot{S}_{in} , and \dot{S}_{tot} when expanded in terms of ϵ , (depending on the model) begins with ϵ^2 or $\dot{\epsilon}$. To be more specific and to elucidate more explicitly our results, let us study the conditions that GSL puts on F in a model whose the Hubble parameter is given by:

$$H = H_0 + \frac{H_1}{t}, \quad H_1 > 0. \quad (24)$$

This universe tends to (quasi-) de Sitter space-time at late time, i.e. when $t^2 \gg \frac{|H_1|}{H_0^2}$. We do not restrict ourselves to one component barotropic matter. By using

$$\begin{aligned} \ddot{F}_R &= F_{RR}\ddot{R} + F_{RG}\ddot{G} + F_{RRR}\dot{R}^2 + 2F_{RRG}\dot{R}\dot{G} + F_{RGG}\dot{G}^2 \\ \ddot{F}_G &= F_{RG}\ddot{R} + F_{GG}\ddot{G} + F_{RRG}\dot{R}^2 + 2F_{RGG}\dot{R}\dot{G} + F_{GGG}\dot{G}^2, \end{aligned} \quad (25)$$

and expanding F_R , F_G , F_{RRR} , and son on around the de Sitter point as what was done in (13), after some computations we obtain

$$\begin{aligned} \dot{S}_h = & -\frac{2\pi H_1}{H_0^2 t^2} \left(12H_0 F_{RR} + 96H_0^3 F_{RG} + 192H_0^5 F_{GG} - \frac{1}{H_0} F_R \right) \\ & + \mathcal{O}\left(\frac{1}{t^3}\right), \end{aligned} \quad (26)$$

and,

$$\begin{aligned} \dot{S}_{tot} = & \frac{48\pi H_1}{H_0^2} (8H_0^2 F_{RG}^2(H_0) + F_{RR}(H_0) + 16H_0^4 F_{GG}(H_0)) \frac{1}{t^3} \\ & + \mathcal{O}\left(\frac{1}{t^4}\right). \end{aligned} \quad (27)$$

Therefore GSL is satisfied provided that

$$8H_0^2 F_{RG}^2(H_0) + F_{RR}(H_0) + 16H_0^4 F_{GG}(H_0) > 0. \quad (28)$$

Note that this result is general, in the sense that it is independent of the explicit form of $F(R, G)$. To go further, let us consider the specific functional form for $F(R, G)$ suggested in the modified Gauss-Bonnet gravity: $F(R, G) = R + g(G)$ where g is a function of G . Viable models are specified by the following conditions [11]: $g(G)$ and its derivatives with respect to G are continuous; $g_{GG} > 0$ for $G < G_0$, where $G_0 = G(H_0)$, and $\lim_{G \rightarrow \infty} g_{GG} = 0$, and finally $0 < H_0^6 g_{GG}(H_0) < 1/384$. Also stability of the model requires that $g_{GG} > 0$ for $G \leq 24H_0^4$. Therefore in this model (28) is satisfied. *It is worthwhile noting that the validity of GSL, in contrast to validity of thermodynamics second law discussed after (16), requires that at early times the universe must be in quintessence phase $\dot{H} < 0$.*

For Einstein theory of gravity we have

$$\dot{S}_{tot} = \frac{2H_1^2}{H_0^5} \frac{1}{t^4} + \mathcal{O}\left(\frac{1}{t^5}\right), \quad (29)$$

and although the thermodynamics second law is only true for $H_1 > 0$, the GSL is valid generally at late time in this model as was expected.

At the end by considering the power law expansion in modified Gauss-Bonnet gravity, we examine the validity of GSL and its consequence on the Hubble parameter. The importance of this example lies on the fact that we can obtain an explicit expression for $F(R, G)$. The accelerated power law expansion of the FRW universe is described by

$$a = a_0 t^m, \quad m > 1, \quad t \geq 0. \quad (30)$$

We also assume that the universe is dominated by a barotropic matter whose equation of state is given by $P_m = w_m \rho_m$. In this model as we use modified gravity there is no need to employ non-ordinary matter.

The Hubble parameter is obtained as $H = \frac{m}{t}$, and the continuity equation yields

$$\rho = \rho_0 t^{-3m\gamma}, \quad (31)$$

where $\gamma = w_m + 1$. We consider a solution to (6) in the form $F = R + g(G)$, where g satisfies the following equation

$$\frac{4}{m-1} \tilde{G}^2 g_{\tilde{G}\tilde{G}} + \tilde{G} g_{\tilde{G}} - g = 6m^2 \tilde{G}^{\frac{1}{2}} - K \tilde{G}^{\frac{3m\gamma}{4}}. \quad (32)$$

We have defined $\tilde{G} := \frac{G}{24m^3(m-1)} = t^{-4}$ and $K = 16\pi\rho_0$. The solution to the above equation is

$$g = C_1 \tilde{G} + C_2 \tilde{G}^{\frac{1-m}{4}} - \frac{4K(m-1)}{(-4+3m\gamma)(-1+(1+3\gamma)m)} \tilde{G}^{\frac{3m\gamma}{4}} - \frac{12m^2(m-1)}{(1+m)} \tilde{G}^{\frac{1}{2}} \quad (33)$$

Nor the field equation neither the thermodynamics second law is affected by the linear term in Gauss-Bonnet invariant, so we set $C_1 = 0$. Authors In [12] argued that for $K = 6m^2$ and $m\gamma = 2/3$ we must have $g = 0$ and therefore $C_2 = 0$. Their argument is based on the fact that in the Einstein theory of gravity we have $K = 6m^2$ and $m\gamma = 2/3$, but we must note that the converse is not true. So although $C_2 = 0$ is a possible choice, it is not necessary, and one can take $C_2 \neq 0$. From (23) one can verify that the GSL is valid when

$$-\frac{1}{3m^3(m-1)}\tilde{G}^{\frac{5}{4}}g_{\tilde{G}\tilde{G}} + \frac{1}{m^2}\tilde{G}^{-\frac{1}{4}} - \frac{K\gamma}{4}\left(\frac{m-1}{m^4}\right)\tilde{G}^{\frac{3}{4}(m\gamma-1)} \geq 0. \quad (34)$$

By putting (33) in (34), we conclude that GSL holds only when

$$\tilde{C}_2 t^{m+1} + \frac{\gamma K}{4} \left(\frac{1}{1 - (1 + 3\gamma)m} + \frac{m-1}{m^2} \right) t^{2-3m\gamma} \leq \frac{1}{m+1}, \quad (35)$$

where $\tilde{C}_2 = \frac{(3+m)C_2}{48m}$. It is clear that an adiabatic expansion is not possible in this model, hence the equality must be excluded in the above relation.

The only free parameter in $F(R, G)$ is C_2 . (35) implies that GSL is always satisfied for $C_2 = 0$ and a FRW universe characterized by

$$0 < m < \frac{2 + 3\gamma - \sqrt{4 + 9\gamma^2}}{6\gamma}, \quad (36)$$

or

$$\frac{1}{1 + 3\gamma} < m < \frac{2 + 3\gamma + \sqrt{4 + 9\gamma^2}}{6\gamma}. \quad (37)$$

So the solution proposed in [12] is in agreement with GSL provided m specified by (30) satisfies (36) or (37). Indeed GSL puts some conditions on the scale factor and therefore on the evolution of the universe.

But it seems that we cannot save this law generally in our method. For example in a model with $m > \frac{2+3\gamma+\sqrt{4+9\gamma^2}}{6\gamma}$ and for ordinary matter $\gamma \geq 1$, GSL does not hold in the limit $t \rightarrow 0$ (more precisely $Kt^{2-3m\gamma} \gg G_N$), or large curvature limit. This may be due to the fact that in our classical computation we have ignored the role of quantum gravity and quantum effects which become important in the large curvature limit (or when $t \rightarrow 0$) or large energy densities.

3 conclusion

In this paper we studied the second and generalized second laws (GSL) of thermodynamics for the apparent horizon in a spatially flat FRW universe in the framework of generalized modified Gauss Bonnet theory of gravity (whose action contains a general function of Gauss Bonnet invariant and

the Ricci scalar: $F(R, G)$). We computed the horizon entropy via Noether charge method (see (9)), and the matter entropy via Friedmann equations to obtain general expressions for the total entropy and its time derivative. We assumed that the horizon temperature which is given by Gibbons-Hawking temperature is the same as the temperature of the matter inside the horizon. It was shown that in the Einstein theory of relativity, the apparent horizon entropy decreases in super-accelerated universe, which may not be the case in $F(R, G)$ theory of gravity. This fact was shown through an example in the "Apparent-horizon entropy" subsection. It must be noted that in the absence of matter or when the contribution of matter entropy is negligible, GSL reduces to the thermodynamics second law for the apparent horizon. It was shown that in an accelerated expanding universe filled with ordinary matter, the matter entropy decreases with time (see (19)) so we must take also into account the contribution of the horizon entropy to get a total entropy which increases with time satisfying GSL. Although in $F(R, G)$ theory of gravity, there may be many choices for $F(R, G)$ which satisfy Friedmann equations and all lead to a same dynamics it was shown that GSL by putting some constraints on $F(R, G)$ can restrict these choices (see (21) and (23)). To elucidate our results we studied GSL in a quasi-de Sitter space time and a universe with power law expansion. We showed that in order that GSL be satisfied in the quasi-de Sitter space time (see (24)) a viable $F(R, G)$ must be chosen (see (28)) which satisfies the stability condition obtained before in the literature but not in the framework of GSL. In the case of power law expansion we obtained an explicit form for $F(R, G)$ and showed that GSL is satisfied provided that the power of time in the scale factor be restricted to some special domain specified by the equation of state parameter of the matter (see (36) and (37)). Despite this, it seems that GSL is violated at large curvature limit which may be due to quantum effects which were ignored during our classical computation.

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